The Substitution Rule

Math 130 - Essentials of Calculus

23 April 2021

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EXAMPLE

Compute the integral

 $2xe^{x^2}dx$

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The Substitution Rule

u-Substitution

The substitution rule is more commonly referred to as "*u*-substitution" because of the following way in which it is usually used:

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$$\int f(\underbrace{g(x)}_{u})\underbrace{g'(x) \, dx}_{du} = \int f(u) \, du = F(u) + C.$$

Math 130 - Essentials of Calculus

EXAMPLES

 $\int x^2 \sqrt{x^3 + 1} \, dx$

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$$\int x^2 \sqrt{x^3 + 1} \, dx$$
 $\int (3x - 2)^{20} \, dx$

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EXAMPLES

$$\int x^2 \sqrt{x^3 + 1} dx$$

$$\int (3x - 2)^{20} dx$$

$$\int t(3 - t^2)^4 dt$$

$$\int \frac{x}{x^2 + 4} dx$$

$$\int x\sqrt{x - 3} dx$$

$$\int \tan x dx$$

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*U***-SUBSTITUTION IN DEFINITE INTEGRALS**

THEOREM (U-SUBSTITUTION FOR DEFINITE INTEGRALS)

If g' is continuous on [a, b] and f is continuous on the range of u = g(x), then

$$\int_a^b f(g(x))g'(x) \ dx = \int_{g(a)}^{g(b)} f(u) \ du.$$

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EXAMPLE

Compute the integrals

$$\int_{0}^{1} \sqrt[3]{1+7x} \, dx$$

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$$\int_{1}^{3} 4z e^{z^{2}-1} dz$$

$$\int_{e}^{e^{4}} \frac{dx}{x \ln x} dx$$

The Substitution Rule