The Substitution Rule

Math 130 - Essentials of Calculus

23 April 2021

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$$\int f(g(x))g'(x)\,dx = F(g(x)) + C.$$

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Example

Compute the integral

$$\int 2xe^{x^2}dx$$

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If $u = g(x)$ is a differentiable function and f is continuous on the range of g, then

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$$\int f(\underbrace{g(x)}_{u})\,\underbrace{g'(x)\,dx}_{du} = \int f(u)\,du = F(u) + C.$$

Examples

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2. $\displaystyle\int (3x-2)^{20}\,dx$
3. $\displaystyle\int t(3-t^2)^4\,dt$
4. $\displaystyle\int \frac{x}{x^2+4}\,dx$
5. $\displaystyle\int x\sqrt{x-3}\,dx$
6. $\displaystyle\int \tan x\,dx$

u-Substitution in Definite Integrals

Theorem (u-Substitution for Definite Integrals)

If g' is continuous on $[a, b]$ and f is continuous on the range of $u = g(x)$, then

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1. $\displaystyle\int_0^1 \sqrt[3]{1+7x}\,dx$
2. $\displaystyle\int_0^2 t^2\sqrt{8-t^3}\,dt$
3. $\displaystyle\int_1^3 4ze^{z^2-1}\,dz$
4. $\displaystyle\int_e^{e^4} \frac{dx}{x\ln x}\,dx$